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Regularization of singular terms in $N\overline{N}$ potential model

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Abstract. We suggest a method of singular terms regularization in potential model of $N\overline{N}$ interaction. This method is free from any uncertainties, related to the usual cut-off procedure and based on the fact, that in the presence of sufficiently strong short-range annihilation N and $\bar{N}$ never approach close enough to each other. The effect of mentioned singular terms of OBE potential, modified by annihilation is shown to be repulsive. The obtained results for S- and P-wave scattering lengths are in agreement with existing theoretical models.

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1 Introduction

During the last decades numerous nonrelativistic models of $N\overline{N}$ low energy interaction [1,2,3,4,5,6,7,8,9,10] have been suggested. An intriguing problem of possible existence of the so called quasi-nuclear $N\overline{N}$ states [2] strongly stimulated the mentioned researches. The physical arguments in favor of such states are the following. The interaction between N and $\overline{N}$ should be much more attractive than the NN interaction, as it follows from the procedure of G-conjugation [1]. Such a strong attraction should produce a spectrum of $N\overline{N}$ quasi-bound states (so called *baryonium*). In the same time the range of annihilation, estimated from the position of the nearest to the threshold singularity in the Feynman annihilation diagrams, is much smaller than the range of the meson exchange forces. This means that baryonium states could be rather narrow to be observed experimentally. It was indeed discovered in the Low Energy Antiproton Ring (LEAR) experiments [12, 13,14] that certain partial cross-sections sharply increase with decreasing of energy down to the $N\overline{N}$ threshold (so called P-wave enhancement), which could be a manifestation of the narrow weakly-bound state or resonance. This conclusion was verified by the experiments with antiprotonic atoms [15,16] and detailed experimental studies of antiproton annihilation at rest by OBELIX collaboration [17,18,19,20]. The recent experimental data [21] on J/ψ decay into $\gamma p\bar{p}$ channel also indicate a strong enhancement near the $p\bar{p}$ threshold.

However, the transparent physical picture of the quasi-nuclear states has a significant drawback. The G-conjugation of NN OBE potential yields in attractive singular terms in $N\overline{N}$ potential of the type $1/r^3$. (In case of NN these terms are repulsive and play a role of the so called short range core). It is well known, that attractive singular potentials produce a collapse [11], i.e. the spectrum of the system is not bounded from below, while the scattering problem has no definite solution. The usual way of dealing with such pathological potentials is to impose, that singular behavior is an artifact of certain approximations (for instance nonrelativistic approximation). In the absence of the self-consistent theory it is common practice to introduce the phenomenological cut-off radius to regularize the singular behavior of the model at short distance. However the results change dramatically with small variations of the cut-off radius (as long as we deal with a real part of $N\overline{N}$ potential) [22] and depend on the details of the cut-off procedure, which seriously diminish the predictive power of the model. In fact it is not clear if the near-threshold states are determined by the "physical part" of the OBEP, or they are artifacts, produced by the "non-physical", singular part of the interaction.

The aim of the present study is to analyze the role of the singular terms and suggest a model of $N\overline{N}$ interaction which is free from the mentioned above uncertainties of the cut-off procedure. The main idea of our approach is that strong enough short range annihilation makes low energy scattering observables independent on any details of the short-range interaction, as far as the particle annihilates rather than "falls to the center".

Mathematically this means that attractive singular potential becomes *regular* when gets the *imaginary* addition to the interaction strength [23]. Curiously this is true even if such an addition is infinitesimal. It is shown that the scattering on the regularized in such a way singular potential is equivalent to the full absorption of the particles in the scattering center. Encouraged by the early result of Dalkarov and Myhrer [3], who introduced a full absorption boundary condition at certain inter-baryonic distance and

successfully described the low energy $N\bar{N}$ scattering data, we suggest a regular potential model of $N\overline{N}$ interaction, based on OBE potential, but without any cut-off radius.

The important feature of our model is that the reflected wave is generated only by those parts of OBE potential, where WKB approximation fails, i.e. by the medium and the long range part of the OBEP. This means that any information about the low energy $N\overline{N}$ scattering could be determined by the mentioned parts of OBEP only. In particular we will show that "singular" part of the potential, modified by annihilation cannot produce any quasi-bound states. The near-threshold resonances, which are well reproduced by our model, are determined by the long range part of OBEP. We calculate the scattering lengths in S- and P- partial waves for different values of spin, isospin and total momentum and demonstrate that obtained results are in good agreement with existing theoretical models [7,8]

The paper is organized as follows. In the second section we discuss the general properties of attractive singular potentials, regularized by imaginary addition to the interaction strength. The third section is devoted to the application of the developed approach to the construction of the regularized $N\overline{N}$ potential model.

2 Inverse power potentials with complex strength.

In this section we present the main results concerning the properties of homogeneous potentials $-\alpha_s/r^s$ with a complex strength $\alpha_s = \operatorname{Re}\alpha_s \pm i\omega$. In the following we put $2M = 1$. Let us first treat the case $s > 2$. Near the origin one can neglect all the terms of the Shrödinger equation, increasing slower than $1/r^2$ and get the following expression for the wave-function [24]:

$$\Phi(r) = \sqrt{r}\left(H_\mu^{(1)}(z) + \exp(2i\delta_0)H_\mu^{(2)}(z)\right) \tag{1}$$

$$z = \frac{2\sqrt{\alpha_s}}{s-2}r^{-(s-2)/2} \tag{2}$$

$$\mu = \frac{2l+1}{s-2} \tag{3}$$

Here $H_\mu^{(1)}(z)$ and $H_\mu^{(2)}(z)$ are the Hankel functions of order μ [25], δ_0 is a contribution of the short range part of the inverse power potential into the scattering phase. It is worth to mention that the variable z is a semiclassical phase.

Let us replace the inverse power potential at distance less than r_0 by the constant $-\alpha_s/r_0^s$, having in mind to tend $r_0 \to 0$. Matching the logarithmic derivatives for the "square-well" solution and the solution (1) at small r_0 , one can get for δ_0:

$$\delta_0 = p(r_0)r_0 \tag{4}$$

$$p(r_0) = \frac{\sqrt{\operatorname{Re}\alpha_s \pm i\omega}}{r_0^{s/2}} \tag{5}$$

Now it is important that the interaction strength α_s is complex. In the limit $r_0 \to 0$ we obtain:

$$\lim_{r_0\to 0}\operatorname{Im}\delta_0 = \operatorname{Im}\frac{\sqrt{\operatorname{Re}\alpha_s \pm i\omega}}{r_0^{(s-2)/2}} \to \pm\infty \tag{6}$$

which means, that $\exp(2i\delta_0)$ is either 0 or ∞ and the linear combination of the Shrödinger equation solutions (1) is uniquely defined in the limit of zero cut-off radius r_0:

$$\lim_{r_0\to 0}\Phi(r) = \begin{cases}\sqrt{r}H_\mu^{(1)}(z) \text{ if } \omega > 0\\ \sqrt{r}H_\mu^{(2)}(z) \text{ if } \omega < 0\end{cases} \tag{7}$$

One can see, that $\omega > 0$ selects an incoming wave , which corresponds to the full absorption of the particle in the scattering center, while $\omega < 0$ selects an outgoing wave, which corresponds to the creation of the particle in the scattering center.

As one can see from (4) and (5) as long as $s > 2$, the above conclusions are valid for any infinitesimal value of ω. It means, that the sign of an infinitesimal imaginary addition to the interaction constant selects the full absorption or the full creation boundary condition (7). This boundary condition can be formulated in terms of the logarithmic derivative in the origin:

$$\lim_{r\to 0}\frac{\Phi'(r)}{\Phi(r)} = -i\,\operatorname{sign}(\omega)p(r) \tag{8}$$

where $p(r)$ is a classical local momentum (5). (Compare with plane incoming (outgoing) wave boundary condition $\exp(\mp ipr)'/\exp(\mp ipr) = \mp ip$).

As soon as the solution of the Shrödinger equation is uniquely defined, we can calculate the scattering observables. In particular we can now obtain the S-wave scattering length for the potential $-(\alpha_s \pm i0)/r^s$ (for $s > 3$) :

$$a = \exp(\mp i\pi/(s-2))\left(\frac{\sqrt{\alpha_s}}{s-2}\right)^{2/(s-2)}\frac{\Gamma((s-3)/(s-2))}{\Gamma((s-1)/(s-2))} \tag{9}$$

The fact, that in spite $\operatorname{Im}\alpha_s \to \pm 0$ the scattering length has nonzero imaginary part is the manifestation of the singular properties of attractive real inverse power potential with $s > 2$ which violates the self-adjointness of the Hamiltonian.

Let us compare the scattering length (9) with that of the repulsive inverse power potential α_s/r^s. One can get:

$$a^{rep} = \left(\frac{\sqrt{\alpha_s}}{s-2}\right)^{2/(s-2)}\frac{\Gamma((s-3)/(s-2))}{\Gamma((s-1)/(s-2))} \tag{10}$$

It is easy to see, that (9) can be obtained from (10) simply by choosing the certain branch (corresponding to an absorption or a creation) of the function $\left(\sqrt{\alpha_s}\right)^{2/(s-2)}$ when passing through the branching point $\alpha_s = 0$. The scattering length in a regularized inverse power potential becomes an analytical function of α_s in the whole complex plane of α_s with a cut along positive real axis. One

can see, that the presence of an inelastic component in the inverse power potential acts in the same way, as a repulsion. It suppresses one of two solutions of the Schrödinger equation and thus eliminates the collapse.

It is easy to see, that the boundary condition (7) of the full absorption (creation) is incompatible with the existence of any bound state. Indeed, one needs both incoming and reflected wave to form a standing wave, corresponding to a bound state. This means, that the regularized inverse power potential supports **no bound states**. This is also clear from the mentioned above fact, that the scattering length for a regularized attractive inverse power potential is an analytical continuation of the scattering length of a repulsive potential.

Let us now turn to the very important case $-\alpha/r^2$. The wave-function now is:

$$\Phi = \sqrt{r}\left[J_{\nu_+}(kr) + \exp(2i\delta_0)J_{\nu_-}(kr)\right] \tag{11}$$

$$\nu_\pm = \pm\sqrt{1/4-\alpha_2} \tag{12}$$

where $k=\sqrt{E}$, and $J_{\nu_\pm}$ are the Bessel functions [25]. In the following we will be interested in the values of $\operatorname{Re}\alpha_2$ greater than critical $\operatorname{Re}\alpha_2 > 1/4$. We use the same cut-off procedure at small r_0. Matching the logarithmic derivatives at cut-off point r_0 we get for $\exp(2i\delta_0)$:

$$\lim_{r_0\to 0}\exp(2i\delta_0) = r_0^{\nu_+-\nu_-} const \sim r_0^{2\nu} = r_0^{\omega/\sqrt{\operatorname{Re}\alpha_2-1/4}} r_0^{-2i\sqrt{\operatorname{Re}\alpha_2-1/4}}$$

One can see, that due to the presence of an imaginary addition ω we get $\operatorname{Im}\delta_0 \to \pm\infty$ when $r_0\to 0$.

Again we come to the boundary condition:

$$\lim_{r_0\to 0}\Phi(r) = \begin{cases} \sqrt{r}J_{\nu_+}(kr) \text{ if } \omega>0 \\ \sqrt{r}J_{\nu_-}(kr) \text{ if } \omega<0 \end{cases} \tag{13}$$

where $\nu_{\pm} = \pm\sqrt{1/4-\alpha_2}$

For the large argument this function behaves like:

$$\Phi \sim \cos(z-\nu_{\pm}\pi/2-\pi/4)$$

The corresponding scattering phase is:

$$\delta = \frac{\pm i\pi}{2}\sqrt{\alpha_2-1/4}+\pi/4 \tag{14}$$

As one can see, the S-matrix $S=\exp(2i\delta)$ is energy independent. This means that the regularized inverse square potential supports **no bound states**. The regularized wave-function and the phase-shift are analytical functions of α_2 in the whole complex plane with a cut along the axis $\operatorname{Re}\alpha_2 > 1/4$.

2.1 Critical strength of inelastic interaction

Now we would like to determine "how strong" should be annihilation to regularize the real attractive singular potential of order s. In other words we would like to find the minimum power t of an *infinitesimal* imaginary inverse power potential required for the regularization of given singular potential. The potential of interest is a sum $-\alpha_s/r^s \mp i\omega/r^t$. Here we keep α_s real. From expressions (4, 5) one immediately comes to the conclusion that the regularization is possible only if:

$$t > s/2+1$$

Thus we come to the conclusion that the scattering is insensitive to any details of the short range modification of a singular interaction if the inelastic component of such an interaction increases in the origin faster than $-1/r^{(s/2+1)}$.

2.2 Singular potential and WKB approximation.

The WKB approximation holds if $|\frac{\partial(1/p)}{\partial r}| \ll 1$. In case of the zero-energy scattering on a regularized inverse power potential with $s>2$ this condition is valid for:

$$r \ll r_{sc} \equiv (2\sqrt{\alpha_s}/s)^{2/(s-2)}$$

(For $s=2$ the semiclassical approximation is valid only for $\alpha_2 \gg 1$). The WKB approximation, consistent with the boundary condition (7) for $s>2$ is :

$$\Phi = \frac{1}{\sqrt{p(r)}}\exp(\pm i\int_r^a p(r)dr) \tag{15}$$

with $p(r)$ from (5). It follows from the above expression, that in case the WKB approximation is valid everywhere the solution of the Schrödinger equation includes incoming wave only (for distinctness we speak here of absorptive potential). The corresponding S-matrix is equal to zero $S=0$ within such an approximation and insensitive to any details of the inner part of potential $p^2(r)$. The outgoing wave can appear in the solution only in the regions where (15) does not hold. For example, in the zero energy limit $k^2\to 0$ the S-matrix is nonzero $S=1-2ika$. One can show that the outgoing wave is reflected from those parts of the potential which change sufficiently fast in comparison with the effective wavelength (so called quantum reflection) $|\frac{\partial(1/p)}{\partial r}|$
*geq*1

For the zero energy scattering and $l=0$ this holds for $r\geq r_{sc}$.

The reflection coefficient $P\equiv|S|^2$ which shows the reflected part of the flux has the following form in the low energy limit:

$$P = 1-4k\operatorname{Im}a$$

For the energies $E \gg E_{sc} \equiv (s/2)^{2s/(s-2)}\alpha_s^{-2/(s-2)}$ the WKB holds everywhere and S-wave reflection becomes exponentially small.

An important conclusion is that any information, which comes from the scattering on an absorptive singular potential is due to a quantum reflection from the tail of such a potential.

2.3 Near-threshold scattering and bound states

Above we have shown that there are no bound states in purely homogenies attractive inverse power potential with complex strength (including the case of infinitesimal imaginary part). The physical reason is the absence of the reflected wave from the absorptive core of inverse power potential. In this subsection we will be interested how the low energy scattering amplitude and spectrum of the near-threshold states of a regular potential $U(r)$ is modified by a potential, which has inverse power behavior $-(\alpha_s+i0)/r^s-\alpha_2/r^2$ near the origin. The cases when such a modification is small are of our special interest.

2.3.1 Penetration under the centrifugal barrier

One could expect that when regularized singular interaction is separated from the regular one by the centrifugal barrier the effect of the regularized singular terms could be small and determined by the centrifugal barrier penetration probability. In fact, if α_s is small enough, there is a range where

$$U(r) \ll \alpha_s/r^s \ll (l(l+1)-\alpha_2)/r^2 \tag{16}$$

Let us suggest that the regular potential is approximately constant in the mentioned range of r, so that $U(r)\approx p^2$. Then from (16) we get:

$$p\alpha_s^{1/(s-2)} \ll 1 \tag{17}$$

For such values of r the wave function is:

$$\Phi \sim \sqrt{r}(J_\mu(pr)-\tan(\delta_s)Y_\mu(pr)) \tag{18}$$

$$Y_\mu = \frac{J_\mu\cos(\mu\pi)-J_{-\mu}}{\sin(\mu\pi)} \tag{19}$$
$$\mu=\sqrt{(l+1/2)^2-\alpha_2}$$

here δ_s is a phase shift, produced by the regularized singular potential in the presence of the regular potential.

For small $r\sim\alpha_s^{1/(s-2)}$ the wave-function is determined by the regularized singular and centrifugal potential:

$$\Phi \sim \sqrt{r}H_\nu^+\left(\frac{2\sqrt{\alpha_s}}{s-2}r^{-(s-2)/2}\right)$$
$$\nu = 2\mu/(s-2)$$

Matching the logarithmic derivatives and taking into account (17) we get for the phase shift δ_s:

$$\delta_s=-\gamma_\mu\left(\frac{p\alpha_s^{1/(s-2)}}{2(s-2)^{2/(s-2)}}\right)^{2\mu}\exp(-i\pi\nu)$$

where

$$\gamma_\mu=\sin(\pi\mu)\frac{\Gamma(1-\mu)\Gamma(1-\nu)}{\Gamma(1+\mu)\Gamma(1+\nu)}$$

Let us mention, that for nonzero l the above expression for $\mathrm{Re}\,\delta_s$ may become inaccurate, as far as the phase shift, produced by the tail of the regularized singular potential, may become greater than the phase shift (**??**), produced by the core of the regularized singular potential. Such a correction depends on the certain form of the tail of the regularized singular potential and can be calculated as a first order of a distorted wave approximation.

In the same time δ_s has positive imaginary part according to the "inelastic" character of regularized singular potential.

$$\mathrm{Im}\,\delta_s=(-1)^l\left(\frac{p\alpha_s^{1/(s-2)}}{2(s-2)^{2/(s-2)}}\right)^{2l+1}\gamma_\mu \tag{20}$$

The near-threshold states produced by the regular potential $U(r)$ are perturbed by the short range regularized singular potential. In particular they get the widths, which in our case of small δ_s are proportional to $\mathrm{Im}\,\delta_s$.

If the near-threshold states spectrum in $U(r)$ has a semiclassical character, than from the quantization rule:

$$\int\sqrt{E_n+\delta E_n-U(r)}dr+\delta_s=const$$

one gets:

$$\delta E_n=-\delta_s\omega_n \tag{21}$$

where ω_n is a semiclassical frequency:

$$\omega_n=\left(\int(E_n-U(r))^{-1/2}dr\right)^{-1}$$

Taking into account (20) we get for the width of the state:

$$\Gamma_n/2=(-1)^{l+1}\left(\frac{p\alpha_s^{1/(s-2)}}{2(s-2)^{2/(s-2)}}\right)^{2l+1}\gamma_\mu\omega_n$$

Thus in the above mentioned case the modification of the near-threshold spectrum of the regular potential $U=p(r)^2$ by the regularized singular potential results in shifting and inelastic broadening determined by the small parameter $(p\alpha_s^{1/(s-2)})^{2l+1}$, which characterizes the centrifugal barrier penetration probability.

2.3.2 Quantum reflection states

We will treat here an interesting case, when there is no barrier separation between (absorptive) regularized singular and regular parts of interaction. However the existence of the narrow near-threshold states is still possible. The reason why in such a case rather narrow states can survive is the so called quantum (over-barrier) reflection from those parts of *attractive* potential, which change sufficiently fast.

To illustrate this idea let us expect that the full interaction potential $U(r)$ has the following form:

$$U(r)=\begin{cases}-(\alpha_s+i0)/r^s & \text{if } r<r_0\\ -(\alpha_s/r_0^s)\Theta(r-R) & \text{if } r\geq r_0\end{cases} \tag{22}$$

This potential can be treated as a shallow and wide square-well with depth α_s/r_0^s and width R perturbed by the regularized singular interaction $-(\alpha_s + i0)/r^s$, which is cut at distance r_0. We are interested in the behavior of the square-well near-threshold spectrum under such a perturbation. For the moment we will restrict our treatment with the S-wave case only.

Let us choose $r_0 > r_{sc} \equiv (2\sqrt{\alpha_s}/s)^{2/(s-2)}$. It was shown above that the WKB approximation, applied to the zero energy scattering on the regularized singular potential fails for $r > r_{sc}$. Thus in our problem there is a domain $r_{sc} < r \leq r_0$ of WKB failure. We will show that this domain acts similar to the barrier in the sense that it is responsible for the reflected wave generation.

For $r \sim r_0$ the zero energy wave-function in the regularized singular potential has the form:

$$\Phi(r) \sim 1 - r/a$$

Here a is the *complex* scattering length (9) in the regularized singular potential $-(\alpha_s + i0)/r^s$. The wave-function in the square well is:

$$\Phi(r) \sim \sin(kr + \delta)$$

Here $k^2 = \alpha_s/r_0^s - E$, where E is the energy of the near-threshold state, while δ is the phase-shift, produced by the regularized singular potential. Matching the logarithmic derivatives at point r_0 and expecting that $|kr_0 + \delta| \ll 1$ we find:

$$\delta = -ka$$

To ensure the existence of the near-threshold state of interest (such that $|k(r_0 - a)| \ll 1$) we choose R big enough. The required characteristic value R_c can be obtained from the condition of the state appearance in the square well of the depth α_s/r_0^s:

$$\sqrt{\alpha_s/r_0^s}R_c = \pi/2$$

Matching the logarithmic derivatives of the square well wave-function and the decaying wave at point R we get for the bound energy $E = -\kappa^2$:

$$k \cot(k(R - a)) = -\kappa$$

which for $\kappa \ll k$ gives:

$$k \simeq \sqrt{\alpha_s/r_0^s} \quad (23)$$

$$\operatorname{Re}\kappa = k^2(R - R_c - \operatorname{Re} a) \quad (24)$$

$$\operatorname{Im}\kappa = -k^2 \operatorname{Im} a \quad (25)$$

The corresponding S-matrix pole in the complex $k-$plane is $z = i\kappa$. One can see, that depending on the sign of $\operatorname{Re}\kappa$ the mentioned pole can be either in the upper half- plane, which corresponds to the bound state, or in the lower half-plane, which corresponds to the virtual state. The width of the state is proportional to the imaginary part of the regularized singular potential scattering length:

$$\Gamma/2 = -k^4 \operatorname{Im} a(R - R_c - \operatorname{Re} a)$$

One can see, that the effect of the regularized singular potential is determined by the parameter $k \operatorname{Im} a$. The physical sense of such a parameter can be easily established. In fact, the S-matrix element corresponding to the scattering with a small momentum k on the regularized singular potential is:

$$S = 1 - 2ika$$

The intensity of the reflected wave is:

$$|S|^2 = 1 - 4k|\operatorname{Im} a|$$

The smaller is $k \operatorname{Im} a$ the higher is the probability of quantum reflection and less is the influence of the regularized singular potential on the near-threshold spectrum of the regular potential.

One can see, that the mentioned above reflection takes place from the domain of WKB failure. (Indeed, as it was shown above in case WKB is valid everywhere the full absorbtion in the origin would ensure that there is no reflected wave, i.e. S-matrix is zero.) This phenomenon of quantum (over-barrier) reflection is known for a long time [27] in different fields, such as neutron physics or physics of ultra-cold atomic collisions [28].

From the above treatment one can get the estimation for the maximum binding energy in a regular potential $U(r)$ modified by an absorptive regularized singular potential.

Let r_0 is the distance where the WKB failure takes place. As we have shown the effect of the WKB failure domain is the partial reflection. Thus, for the purpose of the qualitative estimation, we can replace this domain by the boundary condition of full reflection at r_0. In other words one should look for the bound states in the following truncated potential:

$$U_{tr}(r) = \begin{cases} +\infty \text{ if } r < r_0 \\ U(r) \text{ if } r \geq r_0 \end{cases} \quad (26)$$

The ground state energy E_{tr} in such a potential gives an approximation for the lowest (quasi-)bound state energy in the full potential $U(r)$.

3 Optical model of $N\overline{N}$ interaction.

From the above results it is clear that the model potential, which behaves at short distance like $-(\alpha + i\omega)/r^3$ is regular, i.e. it enables definite unique solution of the scattering problem. Such a potential is absorptive and describes not only elastic, but inelastic scattering as well. The above statements are true even for infinitesimal value of ω. As we have shown such a infinitesimal imaginary addition is equivalent to the full absorption boundary condition in the origin. We apply this regularization procedure to the $N\overline{N}$ potential in ${}^{13}P_0$ state. We use the version of real OBEP potential from the Kohno-Weise model [7] accompanied by the imaginary component $-i\omega/r^3$ with $\omega \to 0$:

$$W = W_{OBEP} - i\omega/r^3$$

State	DR1	DR2	KW	Reg
$^{11}S_0$	0.02-i1.12	0.1-i1.06	-0.03-i1.35	-0.08-i1.16
$^{31}S_0$	1.17-i0.51	1.2-i0.57	1.07-i0.62	1.05-i0.55
$^{13}S_1$	1.16-i0.46	1.16-i0.47	1.24-i0.63	1.19-i0.64
$^{33}S_1$	0.86-i0.63	0.87-i0.67	0.71-i0.76	0.7-i0.65
$^{11}P_1$	-3.33-i0.56	-3.28-i0.78	-3.36-i0.62	-3.19-i0.59
$^{31}P_1$	0.92-i0.5	1.02-i0.43	0.71-i0.47	0.81-i0.46
$^{13}P_0$	-9.58-i5.2	-8.53-i3.51	-8.83-i4.45	-7.67-i4.74
$^{33}P_0$	2.69-i0.13	2.67-i0.15	2.43-i0.11	2.46-i0.15
$^{13}P_1$	5.16-i0.08	5.14-i0.09	4.73-i0.08	4.75-i0.15
$^{33}P_1$	-2.08-i0.86	-2.02-i0.7	-2.17-i0.95	-2.09-i0.79
$^{13}P_2$	0.04-i0.57	0.22-i0.56	-0.03-i0.88	-0.12-i0.82
$^{33}P_2$	-0.1-i0.46	0.05-i0.55	-0.25-i0.39	-0.14-i0.39

Table 1. S- and P-wave scattering lengths

Here W_{OBEP} is the Kohno-Weise real potential **without any cut-off.**

The scattering volume $^3P_0$ $T=0$ calculated in the limit $\omega \to 0$ turns to be $a_r = -7.66 - i4.87$ fm^3, while the value obtained within Kohno-Weise model with a cut-off $r_c = 1$fm is $a_{KW} = -8.83 - i4.45$ fm^3. As one can see, both scattering volumes are rather close. This procedure can be successfully applied to any $N\overline{N}$ state, including attractive singular potential terms.

Obviously, the $N\overline{N}$ states where singular attractive terms are absent do not need any regularization. In this case the inelastic processes can be described by regular imaginary potential, which parameters are carefully fitted for each partial wave. However it is desirable to check if the real OBE potential accompanied with the model of full absorption of the particles in a small volume around the origin can describe the whole set of low energy $N\overline{N}$ scattering data.

We suggest the following modification of the $N\overline{N}$ potential model:

$$W = V_{KW} - i\frac{A}{r^3}\exp(-r/\tau) \qquad (27)$$

here V_{KW} is the real part of Kohno-Weise version of OBE potential [7], but *without any cut-off.* The parameters of the imaginary part of the potential were choused as follows: A=4.7 $GeV\ fm^2$, $\tau = 0.4\ fm$. We have calculated the values of S- and P-scattering lengths in such a model potential. The obtained results, (indicated as Reg) together with the results of two Dover-Richard models, (DR1 and DR2), and Kohno-Weise model (KW) taken from [26] are presented in the Table I. In this table the S-wave scattering lengths are given in fm, while the P-wave scattering volumes are given in fm^3.

One can see rather good agreement between the results obtained within the suggested optical model without cut-off and the cited above versions of Kohno-Weise and Dover-Richard models.

Let us outline here, that there are no reasons to believe that the physical interaction indeed has the above form at small distances. The physical sense of suggested strong annihilation model is that the low energy scattering observables are independent *on any certain form* of short range interaction, and can be obtained by the solution of the Shrödinger equation which is *formally* applied to short distances.

3.1 Near-threshold resonances

The critical question of the quasi-nuclear model is weather the strong annihilation could be compatible with the existence of narrow quasi-bound $N\bar{N}$ states. It follows from the above treatment of our regularized potential model that only few (if any) near-threshold quasi-bound states or resonances can survive, while any deeply bound states are excluded.

We examined the S-matrix poles in the state with the quantum numbers $J=0$, $S=1$, $L=1$, $T=0$. The real and imaginary part of the scattering volume in such a state as they appear in our calculations are very large $a = -7.67 - i4.74\ fm^3$. Such a big value of the scattering volume could be an indication of the near-threshold state or resonance.

Indeed we found that the nearest to the threshold S-matrix poles are situated in the third and fourth quadrant of the complex k-plane:

$$k_+ = 44.8 - i54.3 \text{ MeV/c}\ k_- = -58.4 - i73.8 \text{ MeV/c}$$

In the absence of annihilation such poles should be symmetrical with respect to the imaginary axis and correspond to the near-threshold resonance. The short-range annihilation brakes the left-right symmetry between such poles. Such near-threshold poles will manifest themselves by a rapid increasing of related amplitude and cross-section with the decreasing of energy down to the threshold.

The found resonance belongs to the mentioned above "quantum reflection" case. Indeed, the analysis of our model $N\bar{N}$ potential $U(r)$ (into which we include the centrifugal potential) in the $^{13}P_0$ state shows that the centrifugal barrier is overcome by the attractive singular terms. Thus there is no barrier between regularized singular absorptive core and the "regular" part of the interaction. In the same time one can check that the WKB failure condition:

$$\partial/\partial r(\lambda_{DB}) \geq 1$$

takes place for $r \geq r_0 = 1.4$ fm. Here $\lambda_{DB} = 1/\sqrt{2MU(r)})$ is the local de Broglie wavelength in the case of the zero energy scattering. To judge about the spectrum of $U(r)$ we treat, according to the presented above approach, the potential truncated at point $r_0 = 1.4$ fm (26). One can check that such a potential supports no bound states and the nearest to the threshold S-matrix pole indeed corresponds to the resonance.

Thus the P-wave enhancement is explained in the presented model by the existence of the near-threshold resonance. It should be specially mentioned that no deep bound states with given quantum numbers could exist in our model in spite of strong $N\bar{N}$ attraction.

It is worth to mention that suggested here regularizing procedure can be applied to any OBEP-based optical model of the $N\overline{N}$ low energy interaction.

4 Conclusion

We have found that the scattering observables are insensitive to any details of the short range interaction, if such an interaction includes strong inelastic component and can be found from the solution of the Shrödinger equation, formally applied to short distances. Mathematically this means that singular attractive terms of $N\bar{N}$ potential can be regularized by an imaginary addition to the interaction strength. We analyzed the main properties of such a regularization. In particular it was shown that regularized in mentioned way singular homogenies potential supports no bound states. The low energy scattering amplitude on such a potential is determined by the quantum scattering from the region, where WKB approximation fails (the potential tail). The mentioned formalism was used to build an optical model of $N\bar{N}$ low energy interaction free from uncertainty, related to the cut-off parameter. The good agreement of the results, obtained within our regularization method and within different $N\bar{N}$ interaction models was established. We prove that no deep quasi-bound states are possible within our model, while the low energy scattering observables are determined mainly by the long range part of OBEP. The spectrum of narrow quasinuclear states is concentrated near the threshold. It is argued that the existence of such states is possible due to the phenomenon of quantum (over-barrier) reflection from those parts of $N\bar{N}$ potential which change sufficiently fast. These domains of WKB failure play a role of effective barrier between regular part of interaction and absorptive singular core. In particular, we demonstrate the existence of the near-threshold resonance with quantum numbers $J = 0$, $S = 1$, $L = 1$, $T = 0$, responsible for the P-wave enhancement.

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References

1. R.J.N. Phillips, Rev. Mod. Phys.**39** (1967) 681
2. I.S. Shapiro, Phys. Rep. **35C** (1978) 129
3. O.D. Dalkarov, F. Myhrer, Il Nuovo Cimento **40A** (1977), 152
4. T. Ueda, Prog. Theor. Phys. **62** (1979), 1670
5. J. Cote, M. Lacombe, B. Loiseau, B. Moussallam, R. Vinh Mau, Phys. Rev. Lett.**48** (1982) 1319
6. J.Carbonell, O. Dalkarov, K. Protasov, I. Shapiro, Nucl. Phys.**A535**(1991),651
7. M.Kohno, W. Weise, Nucl. Phys. **A454** (1986) 429
8. Dover, C.B., Richard J.-M.: Phys. Rev. **C21**,1466(1980)
9. M. Pignone, M. Lacombe, B. Loiseau, R. Vinh Mau, Phys. Rev. **C50**(1994), 2710
10. B. El-Bennich, M. Lacombe, B. Loiseau, R. Vinh Mau, Phys.Rev. **C59** (1999) 2313
11. L.D. Landau and E.M. Lifshitz, *Quantum Mechanics*,Pergamon Press, N.Y. 1977
12. A. Martin "Antinucleon-nucleon scattering at LEAR", Proceedings 8th Winter Course, Folgaria, Italy, ed. M. Gibilisco, G. Preparata and A. Zenoni, Singapore: World Scientific (1994)
13. C. Amsler et. al., Phys. Lett. **B297**, 214 (1992)
14. A. Abele et. al., Eur. Phys. J. **C17**, 583 (2000)
15. D. Gotta et. al., Nucl. Phys. **A660** (1999), 283
16. M. Augsburger et. al. Nucl.Phys. **A658** (1999) 149
17. F. Iazzi et. al. Phys. Lett **B475** (2000) 378
18. A. Bianconi et.al. Phys. Lett **B483** (2000)353
19. F. Nichitiu et. al. Phys. Lett **B545** (2002) 261
20. P. Montagna et. al. Nucl. Phys. **A700** (2002) 159
21. J.Z. Bai et. al., Phys. Rev. Lett. 91, 022001 (2003)
22. J. Carbonell, Revista Mexicana de Fisica 47 (2001) 70-77,*Selected topics on Low Energy Antiproton Physics,* e-print arXiv: nucl-th/0103043
23. A. Yu. Voronin, Phys.Rev.A **67**,062706 (2003)
24. M.F. Mott, H.S.W. Massey, *The theory of atomic collisions*, Oxford, At The Clarendon Press, 1965
25. G.N. Watson,*A treatise on the theory of Bessel functions*, Cambridge University Press, 1922
26. J.Carbonell, J.M. Richard, S. Wycech, Z.Phys. A343 (1992) 325
27. J.E. Lennard-Jones, Proc. R. Soc. London, Ser.A **156**,6 (1936)
28. H.Friedrich, G. Jacoby, and C.G. Meister, Phys. Rev. A **65**, 032902